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Experimental investigation of coupled conduction and laminar convection in a circular tube

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Abstract—Wall heat conduction effects on laminar flow heat transfer are experimentally investigated. The steady flow of water through a uniformly heated copper pipe is considered in the experiment, which covers a range of Reynolds numbers from 500 to 1900. The thermal behaviour of the test section is simulated numerically and the influence of conduction along the pipe wall is therefore accounted for in the reduction of the data. Fully developed flow results satisfactorily compare with predictions by a theoretical method previously developed by the authors [*Heat Technol.* **2**, 72 (1984)]. Results are also reported for the case where the velocity profile is partially developed at the inlet of the heat transfer section. The combined effects on heat transfer of flow development and of wall axial heat conduction are discussed.

INTRODUCTION

AXIAL heat conduction along the walls bounding a moving fluid can substantially affect temperature and heat flux distributions in heat transfer devices. To account for such an effect may be necessary on many occasions of practical interest, especially when interpreting experimental data.

Conjugate laminar heat transfer in a circular tube is experimentally investigated in this paper. Measurements are performed for the steady laminar flow of water through a copper pipe whose outer face is uniformly heated. The influence of conduction along the wall on experimental results is estimated by performing a numerical simulation of the test section. Heat transfer results refer to two conditions of the velocity profile at the start of heating. It may either be fully developed, or partially developed if the hydrodynamic entry length is insufficient.

The solution of conjugate heat transfer problems obtained in ref. [1] can apply in the former case. Comparison of experimental results and theoretical predictions therefore complete the analysis.

Previous experimental work on conjugate heat transfer to internal flow is scanty. Several data referring to heat transfer in parallel plate channels have been presented by Mori *et al.* [2–3] for the laminar flow of air and a sodium chloride aqueous solution, and Sakakibara and Endoh [4] for air in turbulent flow. In these experiments the boundary condition imposed at the outer surface of the heating wall was a constant temperature. Counterflow heat exchange was also examined by Mori *et al.* [5]. The experiments of Davis and Cooper [6] dealt with heat transfer to a plane Poiseuille–Couette flow with uniform heat flux at the outer surface of the test section. Theoretical predictions comparable with this experiment were made by Davis and Gill [7]. They served to demonstrate that the increase of the interfacial temperature observed near the leading edge should be attributed to the effect of conduction along the heat transfer section. A few combined conduction and convection results should also be noted: these relate to external flow [8–11].

Coupled conduction and convection in ducts has been considered theoretically by Chu and Bankoff [12] with slug flow, and by Povarnitsyn and Yurlova [13], Luikov *et al.* [14] for Poiseuille flow. In all analyses the duct length was assumed to be infinite or semi-infinite. Laminar flow in pipes and channels with a short heating section has been studied by Shelyag [15] and by Mori *et al.* [2, 3, 16, 17]. Both the conditions of uniform heat flux and of constant temperature at the outer wall surface were considered. The cases of uniform and non-uniform wall thickness for third kind boundary conditions were investigated by the authors [1]. The coupled effect of wall and fluid axial conduction with laminar pipe flow has also received consideration recently [18–20].

The effect of partial flow development on heat transfer in pipes was theoretically investigated by Bankston and McEligot [21], for air. The same problem for highly viscous fluids was considered by Butterworth and Hazell [22] both theoretically and experimentally and by Collins [23]. Axial conduction along the pipe wall was disregarded in all the analyses.

The authors are unaware of any previous experimental data for conjugate heat transfer in circular ducts and of any theoretical or experimental study dealing with wall conduction effects on heat transfer to developing internal flow.

EXPERIMENTAL APPARATUS

The test loop, diagrammatically represented in Fig. 1, had been previously used to investigate the effect of an abrupt convergence on laminar and transitional flow heat transfer. Its general features are described elsewhere [24, 25]. However, certain changes have been made, and the test section as used for this work is

NOMENCLATURE

c	specific heat
h	heat transfer coefficient
k	thermal conductivity
L	effective length of the test section
L_{hy}	hydrodynamic entrance length
L_{hy}^{+}	non-dimensional hydrodynamic entrance length
$\dot{m}$	fluid mass flow rate
Nu_x	local Nusselt number
Pe	Péclet number
Pr	Prandtl number
q	heat transfer rate per unit heat transfer area
q_{gi}	heat generation rate per unit volume
r	radial coordinate
R_0	inner radius of the duct
R_i	outer radius of layer i ($i = 1, 2, 3, 4$)
Re	Reynolds number
T	temperature
U	fluid mean axial velocity
x	axial coordinate
x^*	non-dimensional axial coordinate.

Greek symbols

ε	heat balance error
ξ	dummy variable of the pipe length
ρ	density.

Subscripts

a	external environment
e	initial value at the entrance of the duct
f	fluid
fb	fluid bulk
fd	fully developed laminar flow
out	value at the outlet of the duct
w	fluid–solid interface
x	local value
0	nominal value.

depicted in Fig. 2: it consists of a 10 mm I.D. (18 mm O.D.) copper pipe, 1.0 m long.

Heat is supplied to the fluid along a length of 860 mm via a calibrated constantan wire inserted into a helical groove machined on the outer wall of the pipe. Inner wall surface temperatures are measured by 42 copper–constantan thermocouples along the first 760 mm of the heated section, the remaining part of it acting as a guard. The bulk temperature of the fluid before the hydrodynamic inlet and in the mixing cup following the test section, are measured by two pairs of chromel–alumel thermocouples. The metal pipe is thoroughly insulated along its length by a double sheet of insulating material (asbestos chord and mineral wool).

Flow development at the start of heating was achieved by a hydraulic entrance section of acrylic material upstream of the test section; the entrance length could be varied in steps of 100 mm, from 25 to 325 mm.

Details of the experimental procedure are straightforward and will not be given here. Care was taken in the calibration of the thermocouples and their attachment to the wall so that the overall precision of temperature measurements was estimated to be better

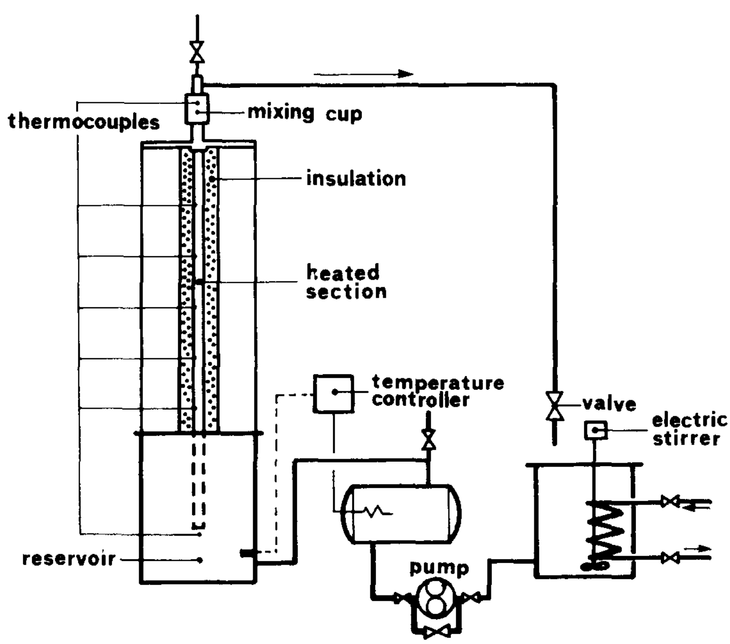

FIG. 1. Schematic diagram of test installation.

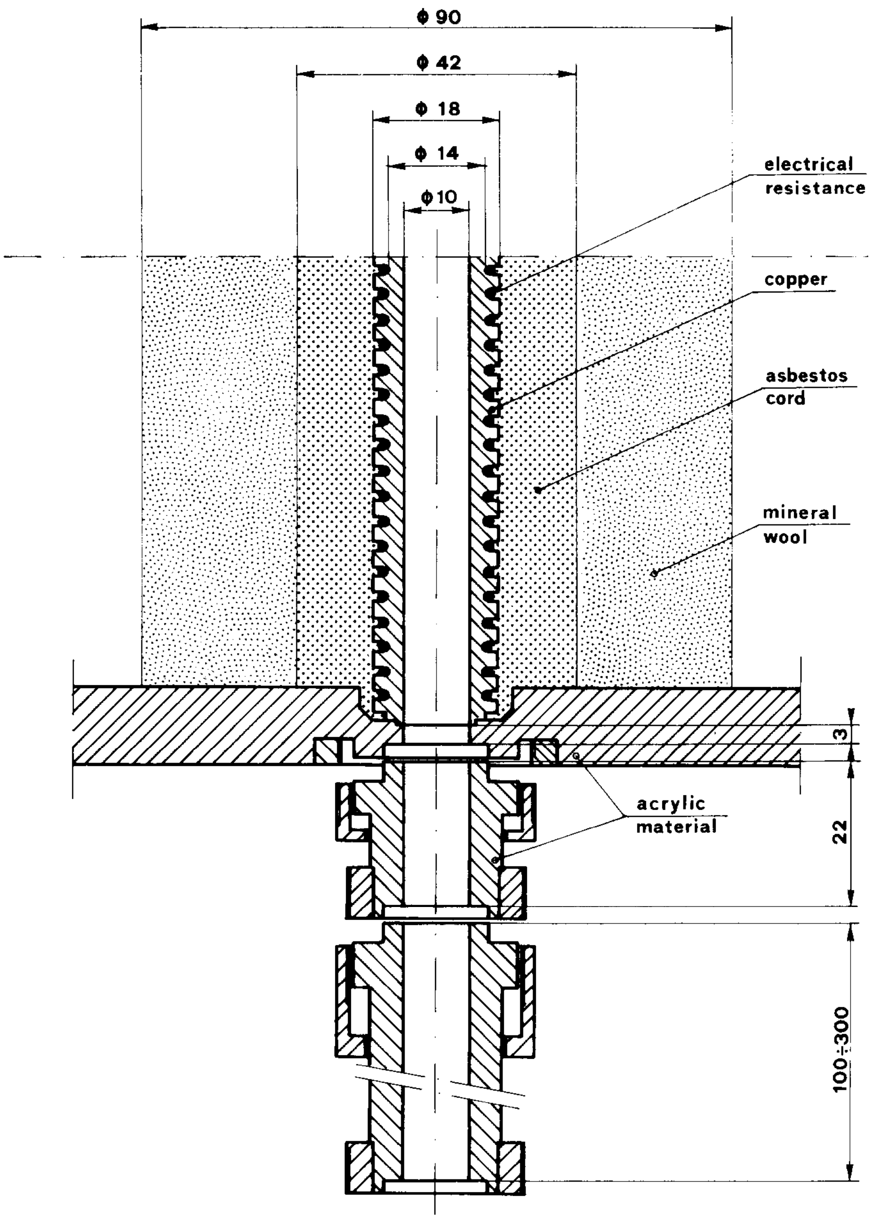

FIG. 2. Detail of the inlet section.

than $\pm 0.1^{\circ}C$. During the course of experimentation fully steady conditions were ensured by repeating temperature readings and flow rate measurements.

Preliminary tests were performed to ascertain the overall accuracy of the measurements in the central portion of the test section, where conductive effects are of minor relevance. The error in the local values of the Nusselt number was estimated to be better than $\pm 5\%$ [26].

NUMERICAL MODELLING OF THE TEST SECTION

In previous experiments [25, 26] a finite-difference method was used to estimate axial heat flow along the wall of the test section. It was assumed that the temperature was uniform and equal to the measured value at any axial position.

Only a rough estimate of the length of the region where axial conduction effects are relevant is nevertheless expected to be achieved in such a way. Uncertainties associated with the determination of the second derivative of the wall temperature and approximations inherent in neglecting dependence of the temperature on the radial coordinate can in fact lead to unpredictable errors near the leading edge.

In the present work the two-dimensional character of the temperature field in the wall is accounted for by a finite-element method developed previously [1]. The physical model of the test section is shown in Fig. 3. Its longitudinal section is discretized with axisymmetric triangular elements assuming the temperature distri-

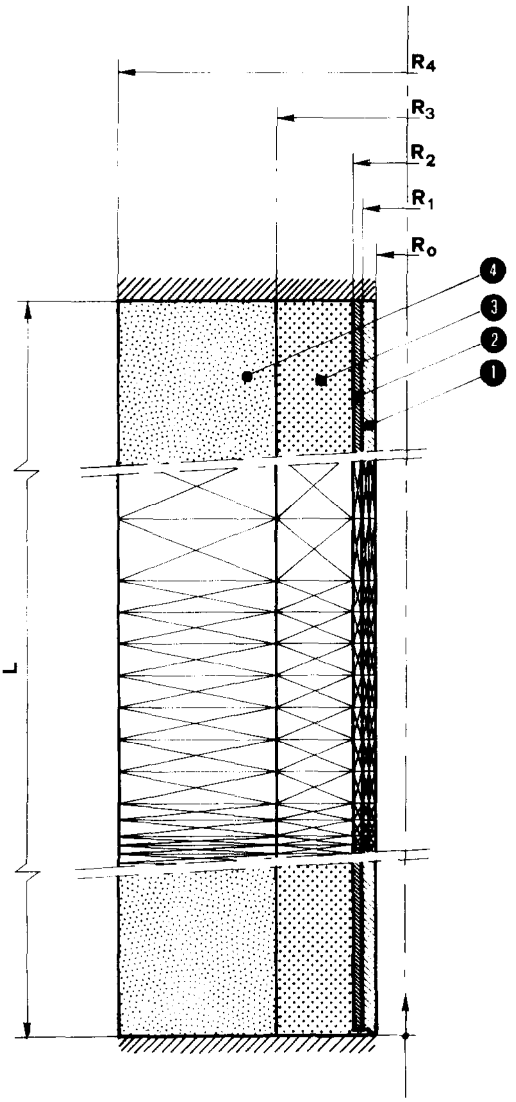

FIG. 3. Physical model of the test section and sample view of the finite-element subdivision.

bution is linear over each element. The part of the wall containing the heating wire is physically inhomogeneous in nature and presents a complex geometry. It is modelled by the substitution of a uniform heat generating layer whose thermal conductivity is taken as 10 W m^{-1} K^{-1}. The lower end of the test section is assumed to be thermally insulated, preliminary tests having shown that heat leakages towards the reservoir cover have a negligible effect on the heat transfer results.

In all the sets performed, experimental values of the wall temperature show a maximum at a downstream distance of about 720 mm from the inlet, due to heat conduction towards the unheated end of the pipe. For convenience, only the length of the heated section extending up to the maximum, L, is considered in the numerical simulation. The section where condition $\mathrm{d}T_w/\mathrm{d}x = 0$ holds is assumed to be thermally insulated. Such a criterion is somewhat arbitrary, nevertheless it has been verified that changing L by ± 50 mm has little effect on the results except in the neighbourhood of $x = L$.

The energy equation is now written in compact form as

$$k_i\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)+q_{gi}=0 \qquad (1)$$

where $i = 1$–4 indicates the layer to be considered as defined by Fig. 3. The heat generation rate per unit volume, q_{gi}, has a non-zero value only for $i = 2$.

Equation (1) is subject to the following set of boundary conditions

$$\frac{\partial T}{\partial x}=0 \quad \text{for} \quad x=L,\ R_0\leqslant r\leqslant R_4 \qquad (2)$$

$$\frac{\partial T}{\partial x}=0 \quad \text{for} \quad x=0,R_0\leqslant r\leqslant R_4 \qquad (3)$$

$$k_4\frac{\partial T}{\partial r}=h_a(T_a-T) \quad \text{for} \quad r=R_4,\ 0\leqslant x\leqslant L \qquad (4)$$

$$k_1\frac{\partial T}{\partial r}=h_x(T-T_{fb}) \quad \text{for} \quad r=R_0,\ 0\leqslant x\leqslant L. \qquad (5)$$

The thermal balance equation at any axial location gives the relationship between the heat flux and the fluid bulk temperature

$$2\pi R_0\int_0^x q_w(\xi)\,\mathrm{d}\xi=\pi\rho c U R_0^2(T_{fb}-T_{f,e}). \qquad (6)$$

In equation (4) h_a is the external heat transfer coefficient. It was necessary to specify this and a value of 7 W m^{-2} K^{-1} was judged to be the best available estimate. In any case the external thermal resistance was only a small part of the total, which was therefore insensitive to the choice of h_a. T_a is the temperature of the external environment, as measured during the course of the experiment. Equation (5) expresses the continuity of the heat flux at the inner boundary of the wall ($r = R_0$).

As h_x in equation (5) is unknown, an iterative scheme is used to solve equations (1)–(6). Guessed values of h_x and T_{fb} are assumed to start the procedure in exactly the same way as outlined in ref. [1] and the wall temperature and heat flux distributions are determined numerically. At any axial position defined by the mesh, new values of T_{fb} and h_x are obtained respectively by equation (6) and the relation

$$h_x=\frac{q_w}{T_w-T_{fb}}. \qquad (7)$$

Here T_w designates interfacial temperature values obtained by splining the experimental values of the wall temperature.

The computing process is repeated until predicted values of the wall temperature at $r = R_0$ differ from T_w by less than ± 0.05°C. Once final values of h_x are achieved, local values of the Nusselt number are computed according to the definition

$$Nu_x=\frac{h_x 2R_0}{k_f}. \qquad (8)$$

Table 1. Data sets of experimental runs

Run	$T_{f,e}$ (°C)	$T_{f,out}$ (°C)	$\dot{m} \times 10^3$ (kg s^{-1})	$q_{w,0}$ (kW m^{-2})	ε (%)	Pr_e	Re_e	L_{hy} (mm)	L^+_{hy}	Pe_e
A △	26.4	29.6	3.6	3.03	4.8	6.16	518	125	0.036	3191
B ▲	22.7	27.2	3.9	3.03	7.1	6.52	525	325	0.062	3423
C **O**	23.4	27.2	4.6	3.04	7.6	6.38	638	325	0.051	4070
D *	20.8	23.6	6.3	3.03	6.8	6.87	821	325	0.040	5640
E ◣	20.1	22.5	7.5	3.03	7.2	7.00	955	325	0.034	6685
F ◺	24.4	27.1	6.8	3.03	3.9	6.20	969	125	0.013	6007
G ■	21.1	24.4	9.2	5.36	7.1	6.82	1203	325	0.027	8204
H □	24.1	27.7	8.7	5.36	5.3	6.26	1230	125	0.010	7700
I ○	24.9	28.4	9.4	5.36	3.9	6.10	1352	125	0.009	8247
L ●	28.9	25.9	10.9	5.36	7.2	6.47	1483	325	0.022	9595
M ★	23.4	25.8	12.6	5.36	9.9	6.37	1748	325	0.018	11 135
N ∅	23.3	25.6	13.6	5.36	8.8	6.40	1864	325	0.017	11 930

It may be worth noting that an iterative scheme is not necessary in principle to solve the problem as the boundary condition, given by equation (5) can be replaced with the simpler statement $T = T_w$ at $r = R_0$. However, the direct imposition of temperature values results in an extremely rigid constraint. Very small irregularities in the axial temperature gradient may lead to oscillations in the Nu_x and q_w distributions near the inlet section.

PROCESSING OF THE EXPERIMENTAL DATA

With distilled water as the working fluid a series of experimental runs was made. Three values of the hydraulic inlet section length were used, namely 125, 225 and 325 mm, and inlet values of the Reynolds number ranged between 300 and 2100. Results are given here only for the 12 runs whose test data are listed in Table 1, other results adding no further information. It is worth pointing out that the heat balance error ε, expressed in Table 1 as a percentage of the electrical power supplied, includes both radial heat losses and leakages from the top end of the pipe. Its values have been reported here for the sake of completeness but have no relevance in the reduction of the data.

Radial heat leakages from $x = 0$ to L were estimated using equation (4) and found to be less than 2% of the average heat flux supplied to the fluid. Relatively low values of heat flux were chosen to reduce the effect on heat transfer of variation in the fluid properties. This aim, however, has not been achieved in the downstream part of the pipe where results are in all probability affected by density and viscosity variations. According to the predictions of Collins [27] for some of the runs reported in ref. [25], such an influence on the Nusselt number values at a downstream distance of 60 diameters should be of the order of 10–15%.

The wall temperature distribution is shown in Fig. 4 for run G. It is representative of the general trend of results obtained for the entire range of these experiments. The interfacial temperature is found to be higher than predicted in the absence of axial wall conduction in the initial part of the test section. It will be noted that $T_w - T_{fb}$ converges to about 4°C at entry. Were axial heat conduction to be insignificant it would, of course, converge to zero. The wall temperature

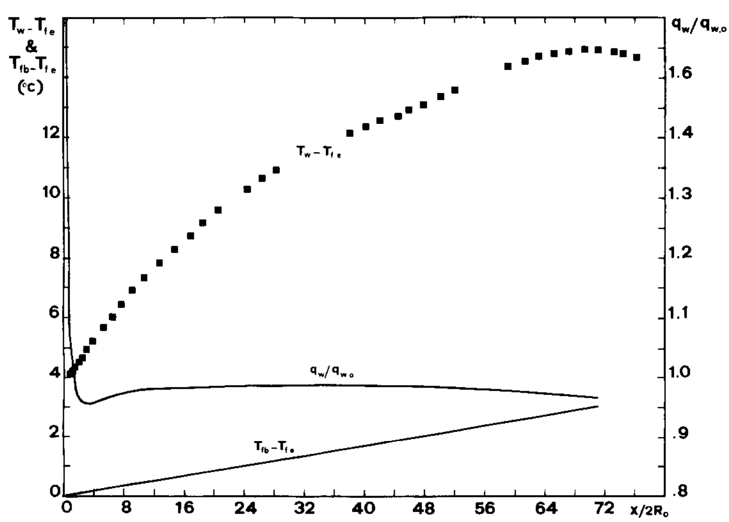

FIG. 4. Measured wall temperature and computed distributions of non-dimensional heat flux and bulk fluid temperature (run G).

exhibits an increasing trend up to about 70 diameters downstream of the inlet section, where T_w reaches a maximum. At this stage, it is desirable to mention the measurements of Polak [28] of wall temperatures for an oil flowing through a small diameter pipe. Subsequently, Martin [29] using an integral profile method, and Collins [30] employing a finite-difference procedure, made comparative predictions of these temperatures. In discussing Martin's results [31] Polak stated that the effect of wall heat conduction would be to reduce predicted temperatures generally, particularly at lower velocities.

Polak's conclusion is surely correct, but the axial distances of wall temperature measurement, and the other experimental conditions cover a different range of parameters to those of the current work.

Inspection of the whole set of data reveals that, in this experiment, gross effects of axial heat conduction on heat transfer are limited to a rather narrow zone near the thermal inlet section. This deduction is confirmed by the inspection of the predicted heat flux distribution. This shows that heat flux density sharply decreases from very high to relatively low values in a downstream distance of the order of two pipe diameters; it then remains lower than the nominal value $q_{w,0}$. Nominal heat flux is defined here as the electrical input per unit area of the inner pipe surface.

It is also pointed out, that deviations of T_{fb} from the linear trend are very small and cannot be detected graphically.

COMPARISON WITH THEORY

The experimental investigation had the objective of corroborating predictions by the theoretical procedure given in ref. [1] for fully developed flow, and also to produce original data on the combined effects of axial wall conduction and velocity development on laminar inlet region heat transfer. Fully developed and developing flow results will, therefore, be discussed separately.

Fully developed flow results

Three main conditions should be satisfied to allow comparison between experimental data and theoretical predictions. The geometrical and thermal characteristics of the test section must be carefully modelled in the analysis, the velocity profile must be fully developed at the thermal inlet section and fluid properties should remain sensibly constant in the experiment.

The first of these conditions is easily satisfied by substituting the four-layer model depicted in Fig. 3 for the homogeneous wall considered in ref. [1]. Due to practical reasons the experimental conditions are more difficult to fulfil. The maximum length of the hydraulic inlet section imposes an upper limit to the inlet Reynolds number value that can be obtained. On the one hand experimental errors become increasingly large with low values of Re_e and heat flux density. However, the deviation from constant property conditions increases if the heating rate is relatively high.

Sets characterized by the highest Reynolds number values consistent with the fully developed flow condition at the inlet section are thus expected to be the most significant for the purpose of comparison with theory. The upper limit for Re_e-values actually depends on the criterion assumed to define the hydrodynamic inlet length, L_{hy}. According to Chen [32] the downstream distance, which is defined as the length where the centre-line velocity reaches 99% of its fully developed value, is determined by

$$\frac{L_{hy,fd}}{2R_0} = \frac{0.60}{0.035Re+1} + 0.056Re. \tag{9}$$

Less strict criteria are nevertheless often suggested in the literature. For instance, according to Porter [33] the relationship

$$L_{hy,fd}/2R_0 = 0.003Re \tag{10}$$

may be used for practical purposes. This corresponds to a 90% centre-line velocity development according to Hornbeck's numerical data [34].

Maximum values of Re_e, for a fully developed velocity profile at $x = 0$, are 580 and 1080 according to equations (9) and (10), respectively. Theoretical predictions have, therefore, been performed for runs B–E, within these limits, and results are compared with experiment in Fig. 5, for two of them.

Overall agreement between theory and measurements is seen to be satisfactory, even if not perfect. Experimental values of the wall temperature are in fact overpredicted and Nu_x-values correspondingly underestimated. The deviations become very small near the inlet section, where axial conduction effects are of greater relevance. They increase with downstream distance due to the growing effect of density and viscosity variation on heat transfer. The general trend of experimental results remains very accurately predicted anyway and this allows the conclusion that theory is sufficiently well confirmed by the experiment.

Developing flow results

For fully developed flow conditions, dimensional analysis considerations indicate that, in the presence of heat conduction along the pipe wall, Nusselt number distribution depends on the wall to fluid conductivity ratio and the Péclet number. Also there are a few parameters which account for the geometrical characteristics of the wall. The Prandtl number enters as an independent quantity when the profile is assumed to develop from a uniform distribution at the start of heating.

In the present case, where in general the velocity profile is neither fully developed nor uniform at $x = 0$, one more parameter is needed to determine the length of the unheated hydrodynamic section in non-dimensional terms. Following Shah and London [35] this has been defined as

$$L^+_{hy} = \frac{L_{hy}}{2R_0\,Re_e} \tag{11}$$

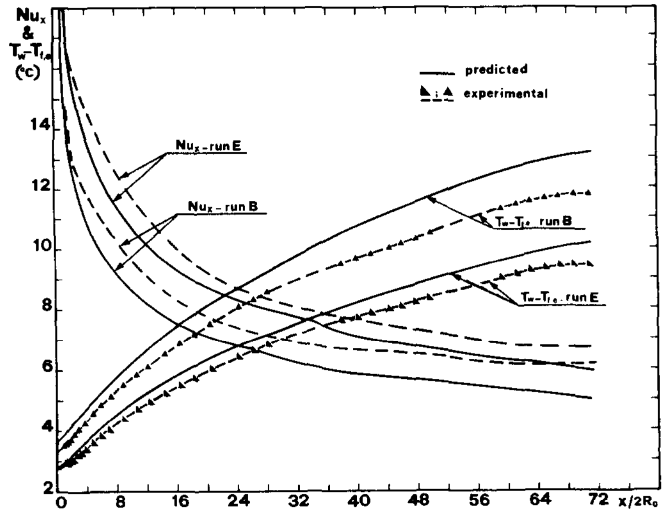

FIG. 5. Comparison of theoretical and experimental wall temperatures and Nusselt numbers (runs B and E).

and its values for the experimental runs presented here are reported in Table 1.

All other relevant quantities remaining almost constant during the course of experimentation, the results demonstrate the influence of the Péclet number and L_{hy}^+ variations (Pe_e = 3200–11 900; L_{hy}^+ = 0.009–0.062) on the heat transfer rate. Nu_x-values for all the runs have been plotted vs $x^* = x/(2R_0\ Pe_e)$ in Fig. 6. Also constant property predictions are shown which omit wall conduction and assume the inlet velocity profile is either parabolic [35], uniform ($Pr = 5$) [36] or partially developed ($Pr = 10$, $L_{hy}^+ = 0.025$ [22]; and $Pr = 10$, $L_{hy}^+ = 0.0025$ [23]). From Fig. 6 it can be observed that Nusselt number distributions exhibit a common behaviour over the entire range of this experiment. Results agree quite closely with uniform wall temperature predictions near the start of heating. After a short distance from the inlet section, whose length reduces for increasing Pe_e-values, transition towards uniform heat flux predictions occurs. Further downstream Nu_x-values then converge to a single line, higher than the predictions assuming constant properties, but having a similar trend. The general shape of the Nu_x-plots is a characteristic effect of the

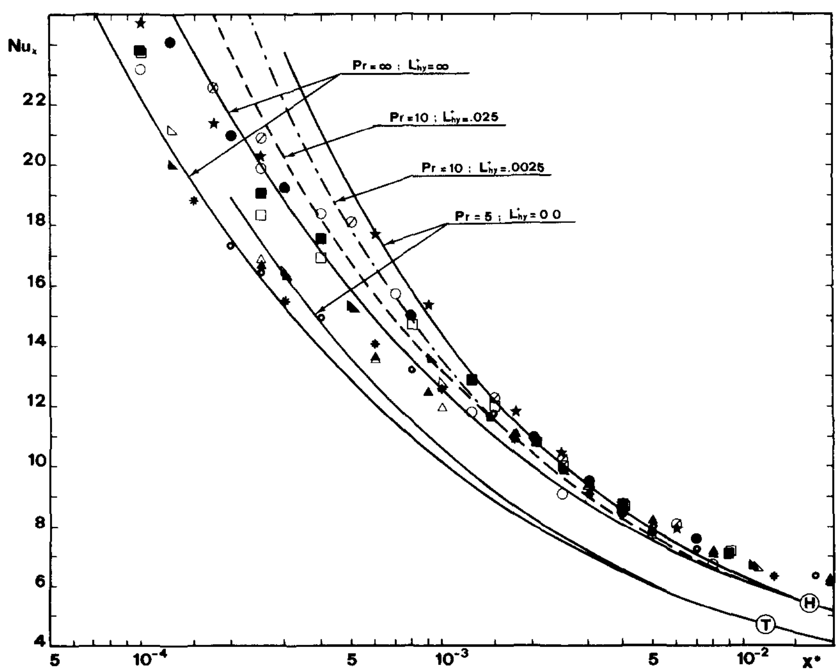

FIG. 6. Experimental Nusselt numbers and theoretical predictions in the absence of axial wall conduction. Uniform wall temperature, case Ⓣ : $Pr = \infty$, $L_{hy}^+ = \infty$ [35]; $Pr = 5$, $L_{hy}^+ = 0$ [36]. Uniform wall heat flux, case Ⓗ : $Pr = \infty$, $L_{hy}^+ = \infty$ [35]; $Pr = 5$, $L_{hy}^+ = 0$ [36]; $Pr = 10$, $L_{hy}^+ = 0.025$ [22]; $Pr = 10$, $L_{hy}^+ = 0.0025$ [23].

presence of conduction along the wall [1, 2, 16]. The range of variability of experimental Nu_x-values is nevertheless much wider than predicted by the theory, especially in the intermediate region, where Nusselt number values become relatively high compared with the reference plots in Fig. 6. This observation leads to the conclusion that entry effects do have a relevance.

While then entry effects seem to be significant in heat transfer terms in this region, the nature of the relationship is not completely clear in the light of current theories. Inspection of the Nu_x-plots in Fig. 6 and consideration of the data sets in Table 1, reveal that the heat transfer rate consistently increases with Péclet number value, as would be expected. However, dependence on L_{hy}^+-values is contradictory. While to reduce non-dimensional hydraulic length by increasing inlet Reynolds number apparently causes an increase in Nu_x-values, shortening the length of the inlet section at the same Re_e-value shows no effect on the heat transfer rate. A comparison of results of runs B, E and G, respectively, with runs A, F and H gives clear experimental evidence of this effect.

Theoretical predictions of Butterworth and Hazell [22] on the effect of incomplete development, however, indicate that the influence of L_{hy}^+ is small over the range covered by these experiments; they then corroborate the conclusion that L_{hy}^+ has little direct influence on the results. This suggests the possibility that heat transfer results might be affected by the onset of some Reynolds-dependent entry effect at relatively high flow rates. Such a hypothesis has actually been forwarded by Collins [30] with regard to some of the experimental results of Butterworth and Hazell [22]. The latter were clearly above numerical heat transfer predictions in the upstream portion of the test section. Direct inspection and replotting of Butterworth and Hazell's data have confirmed that L_{hy}^+ and Re_ϵ did in fact independently influence the heat transfer rate in the experiments.

CONCLUSIONS

The combined effect of laminar convection and axial wall conduction in a uniformly heated pipe have been experimentally investigated. The results serve to complement the theoretical analysis presented in a previous paper and to consider the effect of partial development of inlet velocity profile on laminar flow heat transfer. This latter topic has received very little consideration despite the possibility of practical relevance given that short unheated sections are often used in heat transfer devices.

It is verified that fully developed flow results are well predicted by the theory close to the inlet section where axial conduction effects are decidedly relevant and variation of fluid properties is moderate.

As far as the effect of partial flow development on heat transfer is concerned, results show this to be of minor importance, at least over the relatively narrow range of L_{hy}^+-values explored.

These results are the first experimental evidence of the influence of conduction rate in uniformly heated pipes. The overall data suggest the possibility that, even under laminar flow conditions, entry disturbances may spread a relatively large downstream distance. This effect gives a decided increase in heat transfer rates in the upstream region.

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ETUDE EXPERIMENTALE DE LA CONDUCTION COUPLEE A LA CONVECTION LAMINAIRE DANS UN TUBE CIRCULAIRE

Résumé—On étudie expérimentalement les effets de la conduction dans la paroi sur la convection thermique en l'écoulement laminaire. Un écoulement permanent d'eau à travers un tube de cuivre chauffé uniformément est considéré dans les expériences qui couvre un domaine de nombre de Reynolds allant de 300 à 2100. Le comportement thermique de la section d'essai est numériquement simulé et l'influence de la conduction le long de la paroi du tube est néanmoins considéré dans l'analyse des résultats expérimentaux. Les résultats de l'écoulement pleinement développé s'accordent bien avec les calculs par une méthode théorique antérieurement développée par les auteurs [*Heat Technol.* **2**(1), (1984)]. Des résultats sont aussi donnés pour le cas où le profil de vitesse est partiellement développé à l'entrée de la section thermique. On discute les effets combinés sur le transfert thermique du développement de l'écoulement et de la conduction thermique axiale en paroi.

EXPERIMENTELLE UNTERSUCHUNG DER GEKOPPELTEN WÄRMEÜBERTRAGUNG DURCH LEITUNG UND LAMINARE KONVEKTION IN EINEM KREISRUNDEN ROHR

Zusammenfassung—Der Einfluß von Wärmeleitvorgängen in der Wand auf die Wärmeübertragung bei laminarer Strömung wird experimentell untersucht. Dabei wird die stationäre Strömung von Wasser durch ein gleichmäßig beheiztes Kupferrohr in einem Reynolds-Zahl-Bereich von 300 bis 2100 betrachtet. Das thermische Verhalten der Meßstrecke wird numerisch simuliert, wodurch bei der Versuchsauswertung der Einfluß der axialen Wärmeleitung in der Rohrwand berücksichtigt werden kann. Die Ergebnisse für voll ausgebildete Strömung stimmen befriedigend mit denjenigen Ergebnissen überein, die sich nach einem bereits früher von den Autoren entwickelten theoretischen Verfahren ergeben [*Heat Technol.* **2**(1), (1984)]. Darüber hinaus werden für den Fall eines teilweise ausgebildeten Geschwindigkeitsprofils am Eintritt in den Wärmeübertragungsabschnitt Ergebnisse vorgestellt. Der gekoppelte Einfluß von Strömungsentwicklung und axialier Wärmeleitung in der Wand auf den Wärmeübergang wird diskutiert.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ВЗАИМОСВЯЗАННЫХ ТЕПЛОПРОВОДНОСТИ И ЛАМИНАРНОЙ КОНВЕКЦИИ В КРУГЛОЙ ТРУБЕ

Аннотация—Проведено экспериментальное исследование влияния теплопроводности стенки на теплоперенос при ламинарном течении. Рассмотрено стационарное течение воды в равномерно нагреваемой медной трубе в диапазоне изменения числа Рейнольдса от 300 до 2100. Тепловые характеристики рабочего участка моделируются численно и таким образом при обобщении данных проводится учет влияния на них теплопроводности стенки по длине трубы. Результаты для полностью развитого течения удовлетворительно согласуются с расчетами теоретическим методом, ранее предложенным авторами [см. *Heat Technol.* **2**(1), (1984)]. Также приведены результаты для случая не полностью развитого профиля скорости на входе в тепловой участок. Рассмотрено совместное влияние на теплоперенос развития профиля скорости и осевой теплопроводности стенки.